

Cooper-pair qubit and Cooper-pair electrometer in one device

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An all-superconductor charge qubit enabling a radio-frequency readout of its quantum state is described. The core element of the setup is a superconducting loop which includes the single-Cooper-pair (Bloch) transistor. This circuit has two functions: First, it operates as a charge qubit with magnetic control of Josephson coupling and electrostatic control of the charge on the transistor island. Secondly, it acts as the transducer of the rf electrometer, which probes the qubit state by measuring the Josephson inductance of the transistor. The evaluation of the basic parameters of this device shows its superiority over the rf-SET-based qubit setup.

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Superconducting structures with small Josephson tunnel junctions serve as a basis for electronic devices operating on single Cooper pairs and possessing remarkable characteristics. The paper Ref. [1] has demonstrated the potential of the single-Cooper-pair box circuit [2] as a charge qubit and thus has attracted renewed attention to this field. The practical realization of the Cooper pair qubit is not, however, simple and the main problems here are the achievement of a reliable readout of the quantum state and an elongation of the decoherence time. For the most part, these two issues are interrelated because a charge detector coupled to the qubit presents the principal source of quantum decoherence.

The qubit setup consisting of a *Cooper pair* box and a capacitively coupled single *electron* transistor (SET, including the rf-SET [3]) has been extensively explored [4, 5, 6]. Although a preliminary analysis shows that qubit states can, in principle, be measured in the snapshot regime [6], the ‘mismatch’ of the charge carriers in the setup components (namely, the incoherent nature of charges in the SET) might lead to an unaccounted enhancement of decoherence in the system. Alternatively, the generic type of *Cooper-pair* (Bloch-transistor [7]) electrometers made from superconductors [8, 9] seems to be quite promising as regards matching with the Cooper pair box. Furthermore, similar to dc-SQUIDs [10] and in contrast to SETs, the resistively shunted Cooper-pair electrometer belongs to the category of perfect (quantum-limited) linear detectors [11, 12] and, therefore, can perform continuous measurements of a quantum object [13].

Recently, we have proposed an rf-driven single-Cooper-pair electrometer [14] whose energy-resolution figure ϵ can approach the standard quantum limit of $\hbar/2$. The transducer of this electrometer is a Bloch transistor inserted into a superconducting loop. The magnitude of the supercurrent circulating in the loop depends on the polarization charge (quasicharge) on the transistor island induced via a coupling capacitance by the charge source, e.g., the qubit.

In this paper we present a circuit in which the electrometer’s transducer takes over the function of the Cooper pair box (qubit) as well. The device’s core el-

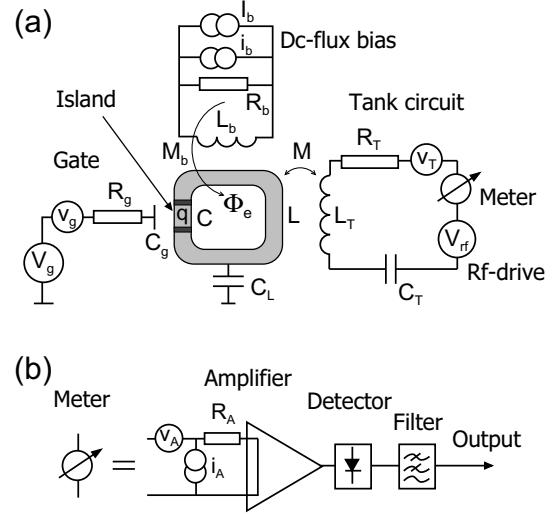


FIG. 1: (a) Electric diagram of the qubit-electrometer system consisting of a macroscopic superconducting low-inductance loop which includes two small Josephson junctions (shown by black color) forming a mesoscopic island in between, equipped with a capacitively coupled gate (the Bloch transistor) and a series-resonance tank circuit inductively coupled to the loop and driven by source V_{rf} . The dc flux Φ_e is applied to the loop by a separate circuit. (b) The rf current meter inserted into the tank circuit is a linear amplifier followed by an amplitude (or phase) detector and a low-pass filter.

ement is a superconducting loop including a mesoscopic double Josephson junction with a capacitive gate (transistor); it is shown in Fig. 1a. The individual junctions are characterized by coupling energies E_{J1} and E_{J2} , which do not differ significantly,

$$k_B T \ll \delta E_J \equiv |E_{J1} - E_{J2}| \ll E_J \simeq E_c, \quad (1)$$

where $E_J \equiv (E_{J1} + E_{J2})/2$ is the average Josephson coupling energy and $E_c \equiv e^2/2C$ is the total charging energy of the island (center electrode) [15]. The island’s total capacitance C is much lower than C_L (the capacitance of the ‘macroscopic’ loop with respect to ground). The

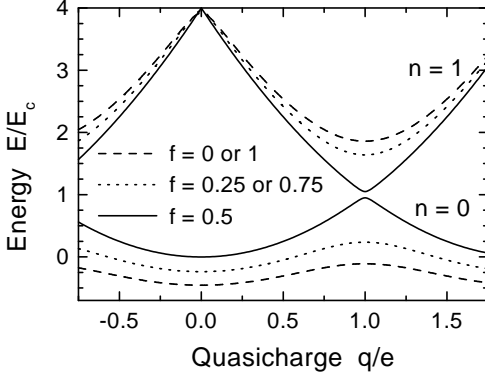


FIG. 2: Energy of the ground ($n = 0$) and upper ($n = 1$) states of the Bloch transistor inserted into the low-inductance superconducting loop at different values of frustration parameter f . The resultant Josephson coupling of the Cooper pair box determines the energy gap in the degeneracy point $q = e$ and varies from $2E_J$ (at $f = 0$) to δE_J (at $f = 0.5$). The circuit parameters are: $E_J = E_c$ and $\delta E_J = 0.1 E_J$.

inductance of the loop L is however rather small,

$$\beta_L = 2\pi L I_c(q) / \Phi_0 \ll 1, \quad (2)$$

where $\Phi_0 = h/2e \approx 2.07$ fWb is the flux quantum. The resulting critical current I_c (a function of the island's quasicharge q) is always lower than its nominal value $I_{c0} = \min\{I_{c1}, I_{c2}\}$, where $I_{c1,c2} = (2\pi/\Phi_0)E_{J1,J2}$, which is realized in the absence of the charging effect (see, e.g., Ref. [11]). Equation (2) ensures a linear relation between the overall Josephson phase, i.e. the sum of individual phases, $\varphi = \varphi_1 + \varphi_2$ (a good quantum variable) and the external magnetic flux Φ_e applied to the loop. Moreover, at small L the characteristic magnetic energy of the loop is $E_m \equiv \Phi_0^2/2L \gg E_J$, and, hence, by far exceeds the energy of thermal fluctuations $k_B T$.

The quasicharge (another good variable) [16], $q = C_g V_g$, is controlled by the voltage V_g applied to a gate with coupling capacitance $C_g \ll C$. A weakly coupled dc-flux-bias circuit fixes the value of the frustration parameter $f = \Phi_e/\Phi_0$ and, hence, the dc Josephson phase $\varphi_0 = 2\pi f$. In particular, the value of $f = 0.5$ makes it possible to significantly reduce the effective Josephson coupling \tilde{E}_J of the system as a Cooper-pair box (qubit), i.e. $\tilde{E}_J = (E_{J1}^2 + E_{J2}^2 + 2E_{J1}E_{J2}\cos\varphi_0)^{1/2} = \delta E_J$, and thereby to localize the region, $q \approx e$, of intensive mixing of the island's charge states $|0\rangle$ and $|2e\rangle$, resulting in the symmetric and antisymmetric basis states: $\frac{1}{\sqrt{2}}(|0\rangle + |2e\rangle)$ at $n = 0$ and $\frac{1}{\sqrt{2}}(|0\rangle - |2e\rangle)$ at $n = 1$ respectively. The corresponding energy spectrum (two lowest Bloch bands) [16] is presented in Fig. 2.

Finally, as shown in the diagram of Fig. 1a, the loop is inductively coupled to a high- Q series tank circuit driven by the sinusoidal voltage V_{rf} of frequency $\omega \approx$

$\omega_0 \equiv (L_T C_T)^{-1/2} \ll \delta E_J/\hbar$. Mutual coupling is relatively weak, $k = M/(LL_T)^{1/2} \ll 1$, while the product $k^2 Q \beta_L$ is not small (~ 1). The change of the amplitude of current oscillations in the tank is amplified, detected, filtered and then serves as an output signal (see Fig. 1b).

The regime analyzed in Ref. [14] assumes a substantial amplitude of induced oscillations of the Josephson phase, $\varphi = \varphi_0 + a \sin \omega t$ with $a \approx 1.8$. This value yields maximum output signals of the rf Bloch electrometer (as well as of the single-junction SQUID operating in the similar, non-hysteretic mode [17]). In this regime, phase φ spans the whole period of 2π ; so the oscillations in the tank in fact probe the critical current of the transistor I_c , whose value depends on the polarization charge on its island q . This charge can be induced, for example, by a charge of a standalone Cooper-pair box coupled to such electrometer via a small capacitance.

In the present case of '100% coupling' between box and electrometer, the amplitude of driving signal V_{rf} can be reduced so that $a \ll 1$. In this regime, the impedance of the low- β_L loop with the Bloch transistor is determined by the Josephson inductance L_J whose reverse value is equal to

$$L_J^{-1}(q, n) = \frac{2\pi}{\Phi_0} \frac{\partial I_s(\varphi, q, n)}{\partial \varphi}. \quad (3)$$

Due to coupling to the loop the effective inductance of the tank is changed,

$$L_T \rightarrow L_T - M^2 L_J^{-1}(q, n), \quad (4)$$

and this leads to the shift of the resonance frequency, $\delta\omega_0/\omega_0 \approx k^2 L L_J^{-1}/2$ [18].

To evaluate this shift we compute the expectation value of the supercurrent operator \hat{I}_s using the Bloch eigenfunctions $|q, n\rangle$ [16] as follows [7],

$$\begin{aligned} I_s(\varphi, q, n) &= \langle q, n | \hat{I}_s | q, n \rangle = \langle q, n | I_{c1} \sin \varphi_1 | q, n \rangle \\ &= \langle q, n | I_{c2} \sin \varphi_2 | q, n \rangle. \end{aligned} \quad (5)$$

The result of numerical calculation for the case of almost symmetric transistor with $E_J = E_c$ is presented in Fig. 3. As can be seen from these plots, not only the maximum of Josephson supercurrent, i.e. the critical current value, depends on the quasicharge q and the band index n [19], but also its phase dependence for $n = 0$ and $n = 1$ is different. In the ground state the current-phase dependence has a shape typical of the Josephson weak links (see, e.g., the review paper [20] and references therein). In the upper state, for all q not close to $0 \bmod(2e)$ and not very large values of the ratio E_J/E_c the dependence has a phase shift of π , which is typical of the Josephson junctions with ferromagnetic interlayer [21].

This behavior of the supercurrent results in the strong dependence of the Josephson inductance (see Eq. (3)) on q and n at $\varphi_0 \approx \pi$, viz., L_J^{-1} is negative in the ground

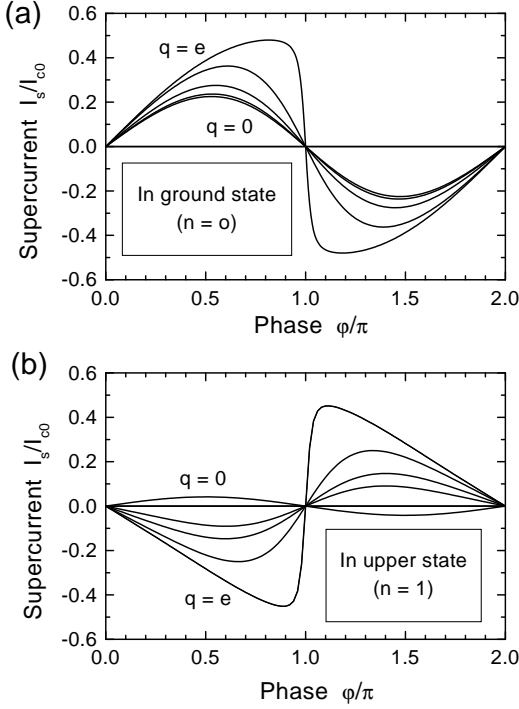


FIG. 3: Supercurrent-phase relation in the Bloch transistor with $E_J = E_c$ and $\delta E_J = 0.1 E_J$ in the ground state [12] (a) and in the upper state (b) calculated in the points of $q = 0, 0.25e, 0.5e, 0.75e$ and e (in the order of increasing amplitude). Note the transformation of the current-phase relation from the regular one to the ‘ π -shifted’ relation, occurring at small q in the upper Bloch state (b).

state and positive in the upper state, while its absolute values $\sim (I_{c0}/\Phi_0)$. Therefore, when the band number n is changing, $0 \rightleftharpoons 1$, the relative shift of the resonance frequency is substantial, i.e.,

$$\delta\omega_0/\omega_0 \sim k^2\beta_L \sim Q^{-1}. \quad (6)$$

Near the resonance this shift leads to a dramatic change of the amplitude of ac current in the tank, $\delta I_a \sim I_a = (\Phi_0/2\pi M)a$, that is reliably measured at $a \gtrsim (\Theta_A B/E_J\omega)^{1/2}$, assuming the output bandwidth $B \lesssim \omega/Q \ll \omega$ and dominating role of the noise of the amplifier whose noise temperature $T_A \equiv \Theta_A/k_B \gg T$ and the input impedance $R_A \gg R_T$ [22].

The great advantage of the superconducting loop including the Bloch transistor consists in a negligibly low dissipation and, therefore, a minor role of intrinsic sources of qubit decoherence [23]. The processes associated with dissipation are the quasiparticle and the pair-quasiparticle interference [24] components of the tunneling. In our case, however, both the frequency and the amplitude of the voltage across the transistor V_{tr} are small, namely $2eV_{tr} = a\hbar\omega \ll \hbar\omega \lesssim k_B T \ll E_c < \Delta$. This relation ensures huge suppression (by a fac-

tor $\eta = \exp(-\Delta/k_B T) \lll 1$) of all dissipative processes. Moreover, since the parity effect blocks sequential quasiparticle tunneling, dissipation can only occur due to the co-tunneling effect. The effective leakage resistance of the transistor (with normal resistances of the junctions R_1 and R_2) therefore is of the order of $(R_1 R_2/R_q)(\Delta/a\eta\hbar\omega)^2 \gg R_{1,2}$, where $R_q = h/4e^2$ is the resistance quantum, and can, therefore, be neglected.

The main source of decoherence therefore is the external electromagnetic circuit: it causes fluctuations of the island’s electric potential \tilde{u} . The fluctuation sources, labeled as v_g , i_b , v_T and v_A in Fig. 1, are associated with dissipative components of the circuit, R_g , R_b , R_T (being presumably at the equilibrium temperature T) and R_A (characterized by T_A). The rates of dephasing caused by the gate and dc-flux bias, i.e. by the sources v_g and i_b respectively, were earlier evaluated by Makhlin, et al. [4]. They showed that reduction of the coupling strengths, i.e., C_g and M_b , can make these rates small enough to allow many single-bit manipulations to be performed within the dephasing time. If $V_{rf} = 0$, similar reasoning can be applied to the resonance tank circuit containing the sources v_T and v_A .

When the rf-drive is on, it leads to an enhancement of the fluctuations \tilde{u} at low frequencies ($\ll \omega$) due to parametric down-conversion of noise at frequencies around ω . The spectral density of these fluctuations [14],

$$S_u(0) \approx a^2 E_J \Theta_A / e^2 \omega, \quad (7)$$

is equivalent to that of the resistance $R_{\text{eff}} = a^2(E_J/\hbar\omega)R_q$ at temperature T_A . At $T_A \approx \hbar\omega/k_B$, the resistance $R_{\text{eff}} \sim R_q/Q$, which allows [4] up to $N = R_q/R_{\text{eff}} \sim Q$ (say, $\sim 10^3$) single-qubit manipulations to be performed in the degeneracy point, $q = e$, during the entanglement time $\tau_{\text{ent}} \sim Q\hbar/\delta E_J$. In contrast to the rf-SET setup [6], there is, in principle, no need for switching the electrometer off in the degeneracy point $q = e$.

During the measurement (away from the point $q = e$) the energy gap between the charge states is large, $\Delta E \equiv \hbar\Omega \gtrsim E_c > \delta E_J$. Due to the high impedance of the tank at the transition frequency $\Omega \gg \omega_0$, the spectral density $S_u(\Omega)$ is remarkably low, viz. $\approx \hbar\omega/\pi Q\Omega L_T$, which yields a long mixing time [5] $\tau_{\text{mix}} = (\hbar/e)^2(\Delta E/E_J)^2 S_u^{-2}(\Omega)$. The latter value allows a measurement with a high signal-to-noise ratio,

$$S/N = (B\tau_{\text{mix}})^{1/2} \sim (\hbar\Omega/k_B T_A)^{1/2} Q \gg 1, \quad (8)$$

to be performed even if a cooled semiconductor amplifier with the fairly good noise temperature of $T_A \sim 10$ K is exploited (cf. $S/N \approx 4$ achievable using an Al rf-SET with extremely low noise figure [6]).

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